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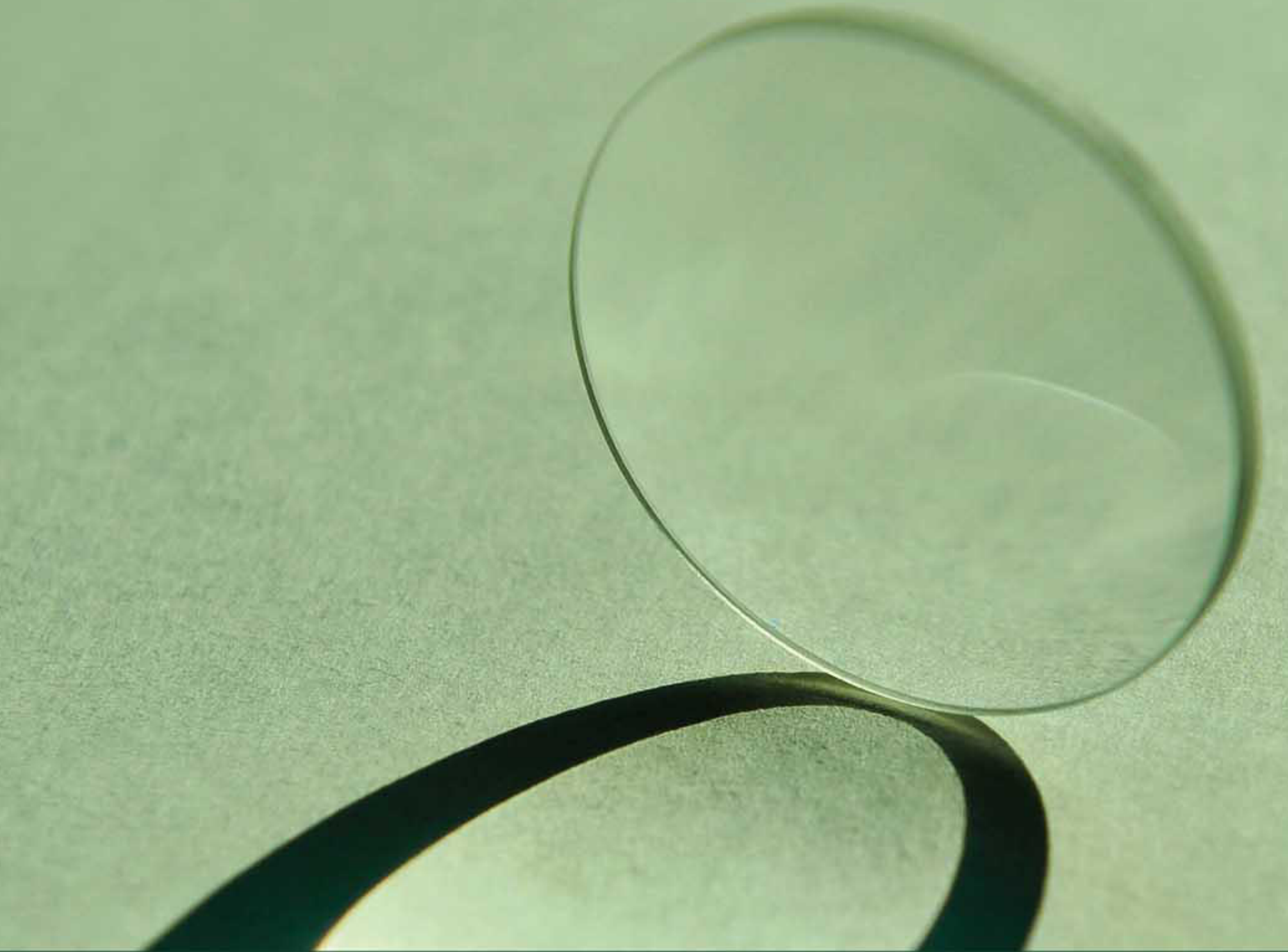

Essential University Physics

Volume 2

FOURTH EDITION

Richard Wolfson

VOLUME TWO

Chapters 20–39

Essential University Physics

FOURTH EDITION

GLOBAL EDITION

Richard Wolfson

Middlebury College

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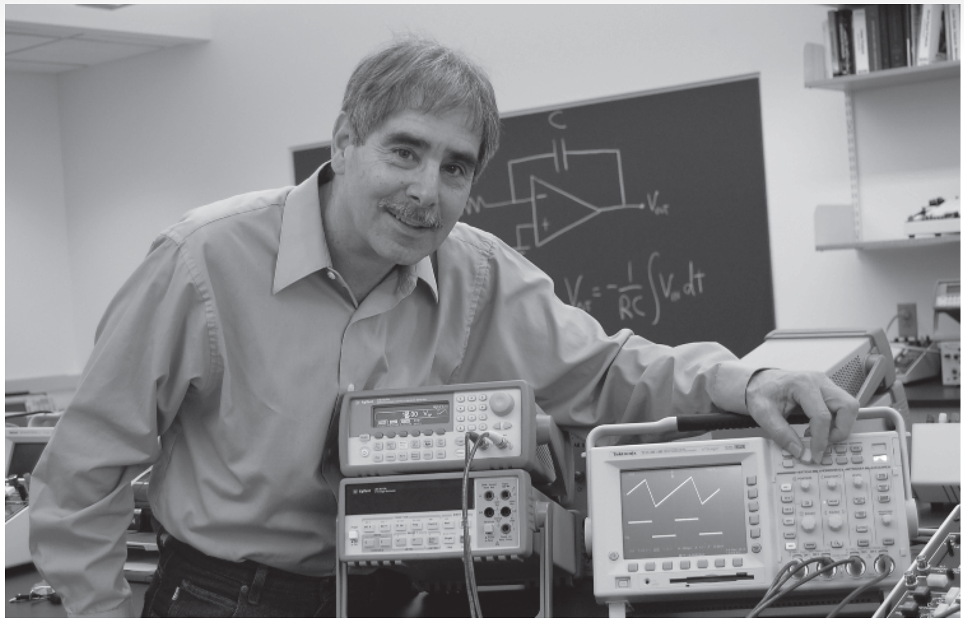
Detailed Contents

About the Author



Richard Wolfson

Richard Wolfson is the Benjamin F. Wissler Professor of Physics at Middlebury College, where he has taught since 1976. He did undergraduate work at MIT and Swarthmore College, and he holds an M.S. from the University of Michigan and a Ph.D. from Dartmouth. His ongoing research on the Sun's corona and climate change has taken him to sabbaticals at the National Center for Atmospheric Research in Boulder, Colorado; St. Andrews University in Scotland; and Stanford University.

Rich is a committed and passionate teacher. This is reflected in his many publications for students and the general public, including the video series *Einstein's Relativity and the Quantum Revolution: Modern Physics for Nonscientists* (The Teaching Company, 1999), *Physics in Your Life* (The Teaching Company, 2004), *Physics and Our Universe: How It All Works* (The Teaching Company, 2011), and *Understanding Modern Electronics* (The Teaching Company, 2014); books *Nuclear Choices: A Citizen's Guide to Nuclear Technology* (MIT Press, 1993), *Simply Einstein: Relativity Demystified* (W. W. Norton, 2003), and *Energy, Environment, and Climate* (W. W. Norton, third edition, 2018); and articles for *Scientific American* and the *World Book Encyclopedia.*

Outside of his research and teaching, Rich enjoys hiking, canoeing, gardening, cooking, and watercolor painting.

Preface to the Instructor

Introductory physics texts have grown ever larger, more massive, more encyclopedic, more colorful, and more expensive. *Essential University Physics* bucks that trend—without compromising coverage, pedagogy, or quality. The text benefits from the author's four decades of teaching introductory physics, seeing firsthand the difficulties and misconceptions that students face as well as the GOT IT? moments when big ideas become clear. It also builds on the author's honing multiple editions of a previous calculus-based textbook and on feedback from hundreds of instructors and students.

Goals of This Book

Physics is the fundamental science, at once fascinating, challenging, and subtle—and yet simple in a way that reflects the few basic principles that govern the physical universe. My goal is to bring this sense of physics alive for students in a range of academic disciplines who need a solid calculus-based physics course—whether they're engineers, physics majors, premeds, biologists, chemists, geologists, mathematicians, computer scientists, or other majors. My own courses are populated by just such a variety of students, and among my greatest joys as a teacher is having students who took a course only because it was required say afterward that they really enjoyed their exposure to the ideas of physics. More specifically, my goals include:

- Helping students build the analytical and quantitative skills and confidence needed to apply physics in problem solving for science and engineering.
- Addressing key misconceptions and helping students build a stronger conceptual understanding.
- Helping students see the relevance and excitement of the physics they're studying with contemporary applications in science, technology, and everyday life.
- Helping students develop an appreciation of the physical universe at its most fundamental level.
- Engaging students with an informal, conversational writing style that balances precision with approachability.

New to the Fourth Edition

The emphasis in this fourth-edition revision has been on pedagogical features, including substantial updates to the end-of-chapter problem sets, learning outcomes, annotated equations, and new, contemporary applications. In addition, I've responded—as I have in previous editions—to the many suggestions made by my colleagues, by instructors around the world, and by reviewers engaged to help make this the most student-friendly and pedagogically useful edition of *Essential University Physics*. And, as always, I've been on the lookout for new developments in physics and technology to incorporate into the text.

- Chapter opening pages have been redesigned to include explicit lists of **learning outcomes** associated with each chapter. Learning outcomes appear at the appropriate section headings and are also keyed with specific problems.
- End-of-chapter problem sets each have over 15% **new problems**. Many of the new problems are of intermediate difficulty, featuring multiple steps and requiring a clear understanding of problem-solving strategies. I've also increased the number of estimation problems and of problems involving symbolic rather than numerical answers. Still other new problems feature contemporary real-world situations.

- Among the most exciting of the new features—and one that gave me both great challenges and great professional satisfaction—are the **Example Variation (EV)** problems. These two sets of four related problems in each chapter, each set based on one of the chapter's worked examples, help the student make connections, enhance her understanding of physics, and build confidence in solving problems different from ones she's seen before. The first problem in each set is essentially the example problem but with different numbers. The second presents the same scenario as the example but asks a different question. The third and fourth problems repeat this pattern but with entirely different scenarios. Working these problems ensures first that the student understands the worked example and then gradually takes her out of her comfort zone to explore new physics, more challenging math, and more complex problem solving.
- Students should perceive a physics textbook as more than a list of equations to consult in solving assigned problems. *Essential University Physics* has always helped students avoid this unfortunate approach to physics. Earlier editions had a few instances where I felt an equation was so important that I developed a separate figure that was essentially an "anatomy" of the equation, with annotations pointing to and explaining the terms in the equation. The new edition extends this approach with **annotated key equations**, giving life to and understanding of all the most important and fundamental equations as statements about the physical universe rather than mere math into which numbers get plugged.
- A host of **new applications** connects physics concepts that students are learning with contemporary technological and biomedical innovations, as well as recent scientific discoveries. A sample of new applications includes the acceleration of striking rattlesnakes, gravitational wave detection and multimessenger astronomy, earthquake resonance effects, the *New Horizons* mission to Pluto, the audacious *Starshot* project, the graded-index lenses of squids' eyes, and environmental and energy issues.
- As with earlier revisions, I've incorporated new research results, new applications of physics principles, and findings from physics education research.
- Finally, this edition includes the 2019 revision of the SI—the international system of units—which represents the most significant change the SI has undergone in more than a century.

Pedagogical Innovations

This book is *concise*, but it's also *progressive* in its embrace of proven techniques from physics education research and *strategic* in its approach to learning physics. Chapter 1 introduces the IDEA framework for problem solving, and every one of the book's subsequent **worked examples** employs this framework. IDEA—an acronym for Identify, Develop, Evaluate, Assess—is not a "cookbook" method for students to apply mindlessly, but rather a tool for organizing students' thinking and discouraging equation hunting. It begins with an interpretation of the problem and an identification of the key physics concepts involved; develops a plan for reaching the solution; carries out the mathematical evaluation; and assesses the solution to see that it makes sense, to compare the example with others, and to mine additional insights into physics. In nearly all of the text's worked examples, the Develop phase includes making a drawing, and most of these use a hand-drawn style to encourage students to make their own drawings—a step that research suggests they often skip. IDEA provides a common approach to all physics problem solving, an approach that emphasizes the conceptual unity of physics and helps break the typical student view of physics as a hodgepodge of equations and unrelated ideas. In addition to IDEA-based worked examples, other pedagogical features include:

- **Problem-Solving Strategy boxes** that follow the IDEA framework to provide detailed guidance for specific classes of physics problems, such as Newton's second law, conservation of energy, thermal-energy balance, Gauss's law, or multiloop circuits.
- **Tactics boxes** that reinforce specific essential skills such as differentiation, setting up integrals, vector products, drawing free-body diagrams, simplifying series and parallel circuits, or ray tracing.

- **QR** codes at the end of each chapter link to resources on Mastering Physics, including video tutorials. These "Pause and predict" videos of key physics concepts ask students to submit a prediction before they see the outcome. The videos are also available in the Study Area of Mastering and in the Pearson eText.
- **GOT IT? boxes** that provide quick checks for students to test their conceptual understanding. Many of these use a multiple-choice or quantitative ranking format to probe student misconceptions and facilitate their use with classroom-response systems.
- **Tips** that provide helpful problem-solving hints or warn against common pitfalls and misconceptions.
- **Chapter openers** that include a graphical indication of where the chapter lies in sequence as well as lists of the learning outcomes and of skills and knowledge needed for the chapter. Each chapter also includes an opening photo, captioned with a question whose answer should be evident after the student has completed the chapter.
- **Applications**, self-contained presentations typically shorter than half a page, provide interesting and contemporary instances of physics in the real world, such as bicycle stability; flywheel energy storage; laser vision correction; ultracapacitors; noise-cancelling headphones; wind energy; magnetic resonance imaging; smartphone gyroscopes; combined-cycle power generation; circuit models of the cell membrane; CD, DVD, and Blu-ray technologies; radiocarbon dating; and many, many more.
- **For Thought and Discussion** questions at the end of each chapter designed for peer learning or for self-study to enhance students' conceptual understanding of physics.
- **Annotated figures** that adopt the research-based approach of including simple "instructor's voice" commentary to help students read and interpret pictorial and graphical information.
- **Annotated equations**, new to the fourth edition, that feature a similar format to the annotated figures.
- **End-of-chapter** problems that begin with simpler exercises keyed to individual chapter sections and ramp up to more challenging and often multistep problems that synthesize chapter material. Context-rich problems focusing on real-world situations are interspersed throughout each problem set.
- **Chapter summaries** that combine text, art, and equations to provide a synthesized overview of each chapter. Each summary is hierarchical, beginning with the chapter's "big ideas," then focusing on key concepts and equations, and ending with a list of "applications"—specific instances or applications of the physics presented in the chapter.

Organization

This contemporary book is *concise*, *strategic*, and *progressive*, but it's *traditional* in its organization. Following the introductory Chapter 1, the book is divided into six parts. Part One (Chapters 2–12) develops the basic concepts of mechanics, including Newton's laws and conservation principles as applied to single particles and multiparticle systems. Part Two (Chapters 13–15) extends mechanics to oscillations, waves, and fluids. Part Three (Chapters 16–19) covers thermodynamics. Part Four (Chapters 20–29) deals with electricity and magnetism. Part Five (Chapters 30–32) treats optics, first in the geometrical optics approximation and then including wave phenomena. Part Six (Chapters 33–39) introduces relativity and quantum physics. Each part begins with a brief description of its coverage, and ends with a conceptual summary and a challenge problem that synthesizes ideas from several chapters.

Essential University Physics is available in two paperback volumes, so students can purchase only what they need—making the low-cost aspect of this text even more attractive. Volume 1 includes Parts One, Two, and Three, mechanics through thermodynamics. Volume 2 contains Parts Four, Five, and Six, electricity and magnetism along with optics and modern physics.

Instructor Supplements

Note: For convenience, all of the following instructor supplements can be downloaded from the Instructor's Resource Area of Mastering™ Physics (www.masteringphysics.com).

Name of Supplement	Instructor or Student Supplement	Description
Mastering™ Physics (www.masteringphysics.com) (9781292350226)	Supplement for Students and Instructors	Mastering Physics from Pearson is the most advanced physics homework and tutorial system available. This online homework and tutoring system guides students through the toughest topics in physics with self-paced tutorials that provide individualized coaching. These assignable, in-depth tutorials are designed to coach students with hints and feedback specific to their individual errors. Instructors can also assign end-of-chapter problems from every chapter, including multiple-choice questions, section-specific exercises, and general problems. Quantitative problems can be assigned with numerical answers and randomized values (with significant figure feedback) or solutions. The Mastering gradebook records scores for all automatically graded assignments in one place, while diagnostic tools give instructors access to rich data to assess student understanding and misconceptions. http://www.masteringphysics.com.
Pearson eText	Supplement for Students and Instructors	The fourth edition of *Essential University Physics* features Pearson eText. The Pearson eText offers students the power to create notes, highlight text in different colors, create bookmarks, zoom, and view single or multiple pages. Professors also have the ability to annotate the text for their course and hide chapters not covered in their syllabi.
Instructor Solutions Manual–Download Only (9781292350196)	Supplement for Instructors	Prepared by John Beetar, the Instructor Solutions Manual contains solutions to all end-of-chapter exercises and problems, written in the Interpret/Develop/Evaluate/Assess (IDEA) problem-solving framework.
Instructor Resources Materials–Download Only (9781292350189)	Supplement for Instructors	Includes all the art, photos, and tables from the book in JPEG format for use in classroom projection or when creating study materials and tests. Also available are downloadable files of the Instructor Solutions Manual and "Clicker Questions," including GOT IT? Clickers, for use with classroom-response systems. Each chapter also has a set of PowerPoint® lecture outlines. These resources are downloadable from the 'Instructor's Resources' area within Mastering Physics. They are also downloadable from the catalog page for Wolfson's *Essential University Physics*, 4th edition, Global Edition at www.pearsonglobaleditions.com
TestGen Test Bank–Download Only (9781292350240)	Supplement for Instructors	The TestGen Test Bank contains more than 2000 multiple-choice, true-false, and conceptual questions. More than half of the questions can be assigned with randomized numerical values.

Acknowledgments

A project of this magnitude isn't the work of its author alone. First and foremost among those I thank for their contributions are the now several thousand students I've taught in calculus-based introductory physics courses at Middlebury College. Over the years your questions have taught me how to convey physics ideas in many different ways appropriate to your diverse learning styles. You've helped identify the "sticking points" that challenge introductory physics students, and you've showed me ways to help you avoid and "unlearn" the misconceptions that many students bring to introductory physics.

Thanks also to the numerous instructors and students from around the world who have contributed valuable suggestions for improvement of this text. I've heard you, and you'll find many of your ideas implemented in this fourth edition of *Essential University Physics*. And special thanks to my Middlebury physics colleagues who have taught from this text and who contribute valuable advice and insights on a regular basis: Jeff Dunham, Mike Durst, Angus Findlay, Eilat Glikman, Anne Goodsell, Noah Graham, Chris Herdmann, Paul Hess, Susan Watson, and especially Steve Ratcliff.

Experienced physics instructors thoroughly reviewed every chapter of this book, and reviewers' comments resulted in substantive changes—and sometimes in major rewrites—to the first drafts of the manuscript. We list these reviewers below. But first, special thanks are due to several individuals who made exceptional contributions to the quality and in some cases the very existence of this book. First is Professor Jay Pasachoff of Williams College, whose willingness more than three decades ago to take a chance on an inexperienced coauthor has made writing introductory physics a large part of my professional career. Dr. Adam Black, former physics editor at Pearson, had the vision to see promise in a new introductory text that would respond to the rising chorus of complaints about massive, encyclopedic, and expensive physics texts. Brad Patterson, developmental editor for the first edition, brought his graduate-level knowledge of physics to a role that made him a real collaborator. Brad is responsible for many of the book's innovative features, and it was a pleasure to work with him. John Murdzek continued Brad's excellent tradition of developmental editing on this fourth edition. We've gone to great lengths to make this book as error-free as possible, and much of the credit for that happy situation goes to John Beetar, who solved every new and revised end-of-chapter problem and updated the solutions manual, and to Edward Ginsberg, who blind-solved all the new problems and thus provided a third check on the answers.

I also wish to thank Nancy Whilton, Jeanne Zalesky, and Tiffany Mok at Pearson Education, and Kim Fletcher at Integra, for their highly professional efforts in shepherding this book through its vigorous production schedule. Finally, as always, I thank my family, my colleagues, and my students for the patience they showed during the intensive process of writing and revising this book.

Reviewers

John R. Albright, *Purdue University–Calumet*
Rama Bansil, *Boston University*
Richard Barber, *Santa Clara University*
Linda S. Barton, *Rochester Institute of Technology*
Rasheed Bashirov, *Albertson College of Idaho*
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Austin Hedeman, *University of California–Berkeley*
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Henry Weigel, *Arapahoe Community College*
Arthur W. Wiggins, *Oakland Community College*
Ranjith Wijesinghe, *Ball State University*
Fredy Zypman, *Yeshiva University*

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Contributors

Aparajita Bandyopadhyay, Ph.D.
Katarzyna Zuleta Estrugo, *École Polytechnique Fédérale de Lausanne*

Reviewers

Michael Cowley, *University of Southern Queensland*
Wayne Rolands, *Swinburne University of Technology*

Preface to the Student

Welcome to physics! Maybe you're taking introductory physics because you're majoring in a field of science or engineering that requires a semester or two of physics. Maybe you're premed, and you know that medical schools are interested in seeing calculus-based physics on your transcript. Perhaps you're really gung-ho and plan to major in physics. Or maybe you want to study physics further as a minor associated with related fields like math, computer science, or chemistry or to complement a discipline like economics, environmental studies, or even music. Perhaps you had a great high-school physics course, and you're eager to continue. Maybe high-school physics was an academic disaster for you, and you're approaching this course with trepidation. Or perhaps this is your first experience with physics. Whatever your reason for taking introductory physics, welcome!

And whatever your reason, my goals for you are similar: I'd like to help you develop an understanding and appreciation of the physical universe at a deep and fundamental level; I'd like you to become aware of the broad range of natural and technological phenomena that physics can explain; and I'd like to help you strengthen your analytic and quantitative problem-solving skills. Even if you're studying physics only because it's a requirement, I want to help you engage the subject and come away with an appreciation for this fundamental science and its wide applicability. One of my greatest joys as a physics teacher is having students tell me after the course that they had taken it only because it was required, but found they really enjoyed their exposure to the ideas of physics.

Physics is fundamental. To understand physics is to understand how the world works, both in everyday life and on scales of time and space so small and so large as to defy intuition. For that reason I hope you'll find physics fascinating. But you'll also find it challenging. Learning physics will challenge you with the need for precise thinking and language; with subtle interpretations of even commonplace phenomena; and with the need for skillful application of mathematics. But there's also a simplicity to physics, a simplicity that results because there are in physics only a very few really basic principles to learn. Those succinct principles encompass a universe of natural phenomena and technological applications.

I've been teaching introductory physics for decades, and this book distills everything my students have taught me about the many different ways to approach physics; about the subtle misconceptions students often bring to physics; about the ideas and types of problems that present the greatest challenges; and about ways to make physics engaging, exciting, and relevant to your life and interests.

I have some specific advice for you that grows out of my long experience teaching introductory physics. Keeping this advice in mind will make physics easier (but not necessarily easy!), more interesting, and, I hope, more fun:

- *Read* each chapter thoroughly and carefully before you attempt to work any problem assignments. I've written this text with an informal, conversational style to make it engaging. It's not a reference work to be left alone until you need some specific piece of information; rather, it's an unfolding "story" of physics—its big ideas and their applications in quantitative problem solving. You may think physics is hard because it's mathematical, but in my long experience I've found that failure to *read* thoroughly is the biggest single reason for difficulties in introductory physics.
- *Look for the big ideas*. Physics isn't a hodgepodge of different phenomena, laws, and equations to memorize. Rather, it's a few big ideas from which flow myriad applications, examples, and special cases. In particular, don't think of physics as a jumble of equations that you choose among when solving a problem. Rather, identify those few big ideas and the equations that represent them, and try to see how seemingly distinct examples and special cases relate to the big ideas.
- *When working problems, re-read* the appropriate sections of the text, paying particular attention to the worked examples. Follow the IDEA strategy described in Chapter 1 and used in every subsequent worked example. Don't skimp on the final Assess step. Always ask: Does this answer make sense? How can I understand my answer in relation to the big principles of physics? How was this problem like others I've worked, or like examples in the text?
- *Don't confuse physics with math*. Mathematics is a tool, not an end in itself. Equations in physics aren't abstract math, but statements about the physical world. Be sure you understand each equation for what it says about physics, not just as an equality between mathematical terms.
- *Work with others*. Getting together informally in a room with a blackboard is a great way to explore physics, to clarify your ideas and help others clarify theirs, and to learn from your peers. I urge you to discuss physics problems together with your classmates, to contemplate together the "For Thought and Discussion" questions at the end of each chapter, and to engage one another in lively dialog as you grow your understanding of physics, the fundamental science.

Video Tutor Demonstrations

Video tutor demonstrations can be accessed by scanning the QR code at the end of each chapter in the textbook using a smartphone. They are also available in the Study Area and Instructor's Resource Area on Mastering Physics and in the eText. Practice Exams and Dynamic Study Modules, which can be used to prepare for exams, are also available in Mastering Physics.

Chapter	Video Tutor Demonstration
2	Balls Take High and Low Tracks
3	Dropped and Thrown Balls
3	Ball Fired from Cart on Incline
3	Ball Fired Upward from Accelerating Cart
3	Range of a Gun at Two Firing Angles
3	Independent Horizontal and Vertical Motion
4	Cart with Fan and Sail
4	Ball Leaves Circular Track
4	Suspended Balls: Which String Breaks?
4	Weighing a Hovering Magnet
5	Tension in String between Hanging Weights
5	Rotational Motion—Loop the Loop
6	Work and Kinetic Energy
7	Chin Basher?
9	Balancing a Meter Stick
9	Water Rocket
9	Happy/Sad Pendulums
9	Conservation of Linear Momentum
10	Canned Food Race
11	Spinning Person Drops Weights
11	Off-Center Collision
11	Conservation of Vector Angular Momentum
11	Conservation of Angular Momentum
12	Walking the Plank
13	Vibrating Rods
13	Resonance of Everyday Items
14	Out-of-Phase Speakers
15	Pressure in Water and Alcohol
15	Water Level in Pascal's Vases
15	Weighing Weights in Water
15	Air Jet Blows between Bowling Balls
15	Bernoulli's Principle—Venturi Tubes
16	Heating Water and Aluminum
16	Water Balloon Held over Candle Flame
16	Candle Chimneys
20	Charged Rod and Aluminum Can
21	Electroscope in Conducting Shell
22	Charged Conductor with Teardrop Shape
23	Discharge Speed for Series and Parallel Capacitors
24	Resistance in Copper and Nichrome
25	Bulbs Connected in Series and in Parallel
26	Magnet and Electron Beam
26	Current-Carrying Wire in Magnetic Field
27	Eddy Currents in Different Metals
29	Parallel-Wire Polarizer for Microwaves
29	Point of Equal Brightness between Two Light Sources
31	Partially Covering a Lens
36	Illuminating Sodium Vapor with Sodium and Mercury Lamps

PART FOUR

Electromagnetism

Electricity constitutes a significant portion of humankind's energy, as evidenced by this composite satellite image of Earth at night. Nearly all that electrical energy is produced by generators, devices that exploit an intimate relation between electricity and magnetism.

OVERVIEW

Electromagnetism is one of the fundamental forces, and it governs the behavior of matter from the atomic scale to the macroscopic world. Electromagnetic technology, from computer microchips to cell phones and on to large electric motors and generators, is essential to modern society. Even our bodies rely heavily on electromagnetism: Electric signals pace our heartbeat, electrochemical processes transmit nerve impulses, and the electric structure of cell membranes mediates the flow of materials into and out of the cell.

Four fundamental laws describe electricity and magnetism. Two deal separately with the two phenomena, while the others reveal profound connections that make electricity and magnetism aspects of a single phenomenon that we call electromagnetism. In this part you'll come to understand those fundamental laws, learn how electromagnetism determines the structure and behavior of nearly all matter, and explore the electromagnetic technologies that play so important a role in your life. Finally, you'll see how the laws of electromagnetism lead to electromagnetic waves and thus help us understand the nature of light.

Electric Charge, Force, and Field

Learning Outcomes

After finishing this chapter you should be able to:

LO 20.1 Describe electric charge as a fundamental property of matter.

LO 20.2 Use Coulomb's law to calculate the forces between charges.

LO 20.3 Use the superposition principle to calculate forces involving multiple charges.

LO 20.4 Describe the concept of electric field.

LO 20.5 Determine the fields of electric charge distributions using superposition.

LO 20.6 Describe the electric dipole and the field it produces.

LO 20.7 Determine the fields of continuous charge distributions by integration.

LO 20.8 Determine the motion of charged particles in electric fields.

LO 20.9 Determine forces and torques on electric dipoles in electric fields.

Skills & Knowledge You'll Need

- The concept of force and Newton's second law (Sections 4.2 and 4.3)
- The gravitational field (Section 8.5)
- Integration techniques for physics (Tactics 9.1)
- The concept of torque, expressed as a cross product (Section 11.2)

What holds your body together? What keeps a skyscraper standing? What holds your car on the road as you round a turn? What governs the electronic circuitry in your computer or smartphone, or provides the tension in your climbing rope? What enables a plant to make sugar from sunlight and simple chemicals? What underlies the awesome beauty of lightning? The answer, in all cases, is the **electric force**. With the exception of gravity, all the forces we've encountered in mechanics—including tension forces, normal forces, compression forces, and friction—are based on electric interactions; so are the forces responsible for all of chemistry and biology. The electric force, in turn, involves a fundamental property of matter—namely, electric charge.

What's the fundamental criterion for initiating a lightning strike?

20.1 Electric Charge

LO 20.1 *Describe electric charge as a fundamental property of matter.*

Electric charge is an intrinsic property of the electrons and protons that, along with uncharged neutrons, make up ordinary matter. What is electric charge? At the most fundamental level we don't know. We don't know what mass "really" is either, but we're familiar with it because we've spent our lives pushing objects around. Similarly, our knowledge of electric charge results from observing the behavior of charged objects.

Charge comes in two varieties, which Benjamin Franklin designated *positive* and *negative*. Those names are useful because the total charge on an object—the object's **net charge**—is the algebraic sum of its constituent charges. Like charges repel, and opposites attract, a fact that constitutes a qualitative description of the electric force.

Quantities of Charge

All electrons carry the same charge, and all protons carry the same charge. The proton's charge has *exactly* the same magnitude as the electron's, but with opposite sign. Given that electrons and protons differ substantially in other properties—like mass—this electric relation is remarkable. Exercise 11 shows how dramatically different our world would be if there were even a slight difference between the magnitudes of the electron and proton charges.

The magnitude of the electron or proton charge is the **elementary charge** e. Electric charge is **quantized**; that is, it comes only in discrete amounts. In a famous experiment in 1909, the American physicist R. A. Millikan measured the charge on small oil drops and found it was always a multiple of a basic value we now know as the elementary charge.

Elementary particle theories show that the fundamental charge is actually $\frac{1}{3}e$. Such "fractional charges" reside on quarks, the building blocks of protons, neutrons, and many other particles. Quarks always join to produce particles with integer multiples of the full elementary charge, and it seems impossible to isolate individual quarks.

The SI unit of charge is the **coulomb** (C), named for the French physicist Charles Augustin de Coulomb (1736–1806). From the late 19th century to the early 21st century, the coulomb was defined in terms of electric current and time—a definition that was difficult to implement in practice. The 2019 revision of the SI gave the coulomb a much simpler definition. Now, the elementary charge is defined to be exactly $1.602176634 \times 10^{-19}$ C. The coulomb is therefore the number of elementary charges equal to the inverse of this number. For our purposes, that's about 6.24×10^{18} elementary charges.

Charge Conservation

Electric charge is a conserved quantity, meaning that the net charge in a closed region remains constant. Charged particles may be created or annihilated, but always in pairs of equal and opposite charge. The net charge always remains the same.

GOT IT?

20.1 The proton is a composite particle composed of three quarks, all of which are either *up quarks* (u; charge $+\frac{2}{3}e$) or *down quarks* (d; charge $-\frac{1}{3}e$). (More on quarks in Chapter 39.) Which of these quark combinations is the proton? (a) *udd*; (b) *uuu*; (c) *uud*; (d) *ddd*

20.2 Coulomb's Law

LO 20.2 *Use Coulomb's law to calculate the forces between charges.*

LO 20.3 *Use the superposition principle to calculate forces involving multiple charges.*

Rub a balloon; it gets charged and sticks to your clothing. Charge another balloon, and the two repel (Fig. 20.1). Socks cling to your clothes as they come from the dryer, and bits of Styrofoam cling annoyingly to your hands. Walk across a carpet, and you'll feel a shock when you touch a doorknob. All these are common examples where you're directly aware of electric charge.

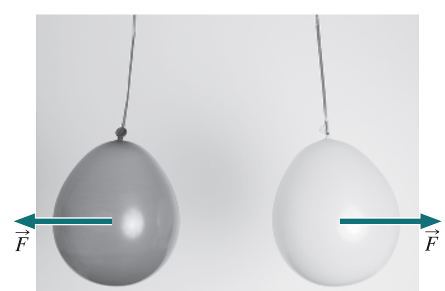


FIGURE 20.1 Two balloons carrying similar electric charges repel each other.

Electricity would be unimportant if the only significant electric interactions were these obvious ones. In fact, the electric force dominates all interactions of everyday matter, from the motion of a car to the movement of a muscle. It's just that matter on a large scale

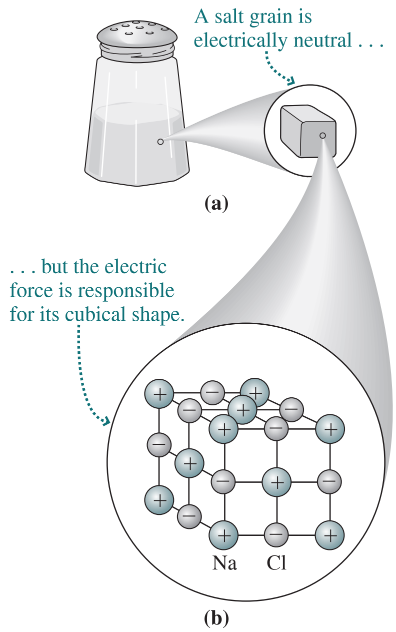


FIGURE 20.2 (a) A single salt grain is electrically neutral, so the electric force isn't obvious. (b) Actually, the electric force determines the structure of salt.

is almost perfectly neutral, meaning it carries zero net charge. Therefore, electric effects aren't obvious. But at the molecular level, the electric nature of matter is immediately evident (Fig. 20.2).

Attraction and repulsion of electric charges imply a force. Joseph Priestley and Charles Augustin de Coulomb investigated this force in the late 1700s. They found that the force between two charges acts along the line joining them, with the magnitude proportional to the product of the charges and inversely proportional to the square of the distance between them. **Coulomb's law** summarizes these results:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r} \quad \text{(Coulomb's law)} \qquad (20.1)$$

$\vec{F}_{12}$ is the force charge q_1 exerts on charge q_2.

k is approximately $9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$.

q_1 and q_2 are two charges.

r is the distance between the two charges.

$\hat{r}$ is a unit vector that points from q_1 toward q_2 regardless of the signs of the charges.

where $\vec{F}_{12}$ is the force charge q_1 exerts on q_2 and r is the distance between the charges. In SI the proportionality constant k has the approximate value $9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$. Force is a vector, and $\hat{r}$ is a unit vector that helps determine its direction. Figure 20.3 shows that $\hat{r}$ lies on a line passing through the two charges and points in the direction *from q_1 toward q_2*. Reverse the roles of q_1 and q_2, and you'll see that $\vec{F}_{21}$ has the same magnitude as $\vec{F}_{12}$ but the opposite direction; thus Coulomb's law obeys Newton's third law. Figure 20.3 also shows that the force is in the same direction as the unit vector when the charges have the same sign, but opposite the unit vector when the charges have different signs. Thus Coulomb's law accounts for the fact that like charges repel and opposites attract.

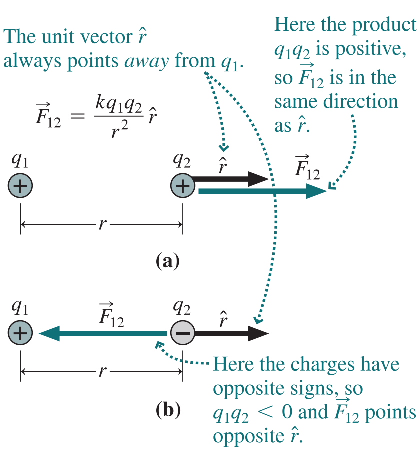


FIGURE 20.3 Quantities in Coulomb's law for calculating the force $\vec{F}_{12}$ that q_1 exerts on q_2.

PROBLEM-SOLVING STRATEGY 20.1 Coulomb's Law

The key to using Coulomb's law is to remember that force is a vector, and to realize that Coulomb's law in the form of Equation 20.1 gives both the magnitude and direction of the electric force. Dealing carefully with vector directions is especially important in situations with more than two charges.

INTERPRET First, make sure you're dealing with the electric force alone. Identify the charge or charges on which you want to calculate the force. Next, identify the charge or charges producing the force. These comprise the **source charge**.

DEVELOP Begin with a drawing that shows the charges, as in Fig. 20.4. If you're given charge coordinates, place the charges on the coordinate system; if not, choose a suitable coordinate system. For each source charge, determine the unit vector(s) in Equation 20.1. If the charges lie along or parallel to a coordinate axis, then the unit vector will be one of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, or $\hat{k}$, perhaps with a minus sign. In Fig. 20.4, the force on q_3 due to q_1 is such a case. When the two charges don't lie on a coordinate axis, like q_1 and q_2 in Fig. 20.4, you can find the unit vector by noting that the displacement vector $\vec{r}_{12}$ points in the desired direction, from the source charge to the charge experiencing the force. Dividing $\vec{r}_{12}$ by its own magnitude then gives the unit vector in the direction of $\vec{r}_{12}$; that is, $\hat{r} = \vec{r}_{12}/r_{12}$.

EVALUATE For each source charge, determine the electric force using Equation 20.1,

$$\vec{F}_{12} = (kq_1q_2/r^2)\hat{r}$$

with $\hat{r}$ the unit vector you've just found.

ASSESS As always, assess your answer to see that it makes sense. Is the direction of the force you found consistent with the signs and placements of the charges giving rise to the force?

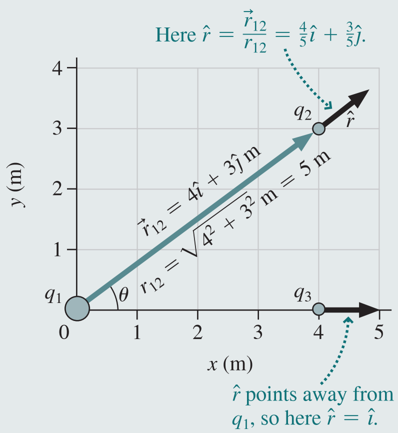


FIGURE 20.4 Finding unit vectors.

GOT IT?

20.2 Charge q_1 is located at $x = 1$ m, $y = 0$. What should you use for the unit vector $\hat{r}$ in Coulomb's law if you're calculating the force that q_1 exerts on a charge q_2 located (1) at the origin and (2) at the point $x = 0$, $y = 1$ m? Explain why you can answer without knowing the sign of either charge.

EXAMPLE 20.1 Finding the Force: Two Charges

A 1.0-μC charge is at $x = 1.0$ cm, and a −1.5-μC charge is at $x = 3.0$ cm. What force does the positive charge exert on the negative one? How would the force change if the distance between the charges tripled?

INTERPRET Following our strategy, we identify the −1.5-μC charge as the one on which we want to find the force and the 1-μC charge as the source charge.

DEVELOP We're given the coordinates $x_1 = 1.0$ cm and $x_2 = 3.0$ cm. Our drawing, Fig. 20.5, shows both charges at their positions on the x-axis. With the source charge q_1 to the left, the unit vector in the direction from q_1 toward q_2 is $\hat{\imath}$.

EVALUATE Now we use Coulomb's law to evaluate the force:

$$\vec{F}_{12} = \frac{kq_1q_2}{r^2}\hat{r}$$

$$= \frac{(9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0 \times 10^{-6}\ \text{C})(-1.5 \times 10^{-6}\ \text{C})}{(0.020\ \text{m})^2}\hat{\imath}$$

$$= -34\hat{\imath}\ \text{N}$$

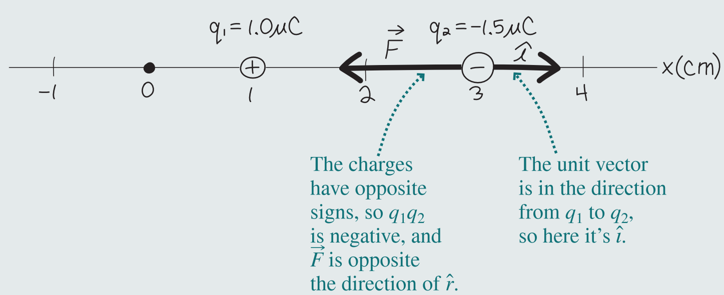


FIGURE 20.5 Sketch for Example 20.1.

This force is for a separation of 2 cm; if that distance tripled, the force would drop by a factor of $1/3^2$, to $-3.8\hat{\imath}$ N.

ASSESS Make sense? Although the unit vector $\hat{\imath}$ points in the $+x$-direction, the charges have opposite signs and that makes the force direction opposite the unit vector, as shown in Fig. 20.5. In simpler terms, we've got two opposite charges, so they attract. That means the force exerted on a charge at $x = 3$ cm by an opposite charge at $x = 1$ cm had better be in the $-x$-direction.

CONCEPTUAL EXAMPLE 20.1 Gravity and the Electric Force

The electric force between elementary particles is far stronger than the gravitational force, yet gravity is much more obvious in everyday life. Why?

EVALUATE Gravity and the electric force obey similar inverse-square laws, and the magnitude of the force is proportional to the product of the masses or charges. There's a big difference, though: There's only one kind of mass, and gravity is always attractive, so large concentrations of mass—like a planet—result in strong gravitational forces. But charge comes in two varieties, and opposites attract, so large accumulations of matter tend to be electrically neutral, in which case large-scale electrical interactions aren't obvious.

ASSESS Ironically, it's the very strength of the electric force that makes it less obvious in everyday life. Opposite charges bind strongly, making bulk matter electrically neutral and its electrical interactions subtle.

MAKING THE CONNECTION Compare the magnitudes of the electric and gravitational forces between an electron and a proton.

EVALUATE Equation 8.1 gives the gravitational force: $F_g = Gm_e m_p/r^2$. Equation 20.1 gives the electric force: $|F_E| = ke^2/r^2$, where we wrote e^2 because the electron and proton charges have the same magnitude. We aren't given the distance, but that doesn't matter because both forces have the same inverse-square dependence. The ratio of the force magnitudes is huge: $F_E/F_g = ke^2/Gm_e m_p = 2.3 \times 10^{39}$!

Point Charges and the Superposition Principle

Coulomb's law is strictly true only for **point charges**—charged objects of negligible size. Electrons and protons can usually be treated as point charges; so, approximately, can any two charged objects if their separation is large compared with their size. But often we're interested in the electric effects of **charge distributions**—arrangements of charge spread over space. Charge distributions are present in molecules, memory cells in your computer, your heart, and thunderclouds. We need to combine the effects of two or more charges to find the electric effects of such charge distributions.

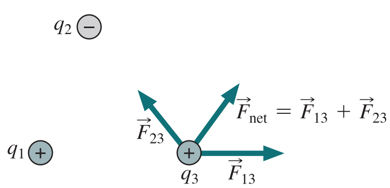


FIGURE 20.6 The superposition principle lets us add vectorially the forces from two or more charges.

Figure 20.6 shows two charges q_1 and q_2 that constitute a simple charge distribution. We want to know the net force these exert on a third charge q_3. To find that net force, you might calculate the forces $\vec{F}_{13}$ and $\vec{F}_{23}$ from Equation 20.1, and then vectorially add them. And you'd be right: The force that q_1 exerts on q_3 is unaffected by the presence of q_2, and vice versa, so you can apply Coulomb's law separately to the pairs q_1q_3 and q_2q_3 and then combine the results. That may seem obvious, but nature needn't have been so simple.

The fact that electric forces add vectorially is called the **superposition principle**. Our confidence in this principle is ultimately based on experiments showing that electric and indeed electromagnetic phenomena behave according to the principle. With superposition we can solve relatively complicated problems by breaking them into simpler parts. If the superposition principle didn't hold, the mathematical description of electromagnetism would be far more complicated.

Although the force that one point charge exerts on another decreases with the inverse square of the distance between them, the same is not necessarily true of the force resulting from a charge distribution. The next example provides a case in point.

EXAMPLE 20.2 Finding the Force: Raindrops
Worked Example with Variation Problems

Charged raindrops are ultimately responsible for lightning, producing substantial electric charge within specific regions of a thundercloud. Suppose two drops with equal charge q are on the x-axis at $x = \pm a$. Find the electric force on a third drop with charge Q at an arbitrary point on the y-axis.

INTERPRET Coulomb's law and the superposition principle apply, and we identify Q as the charge for which we want the force. The two charges q are the source charges.

DEVELOP Figure 20.7 is our drawing, showing the charges, the individual force vectors, and their sum. The drawing shows that the distance r in Coulomb's law is the hypotenuse $\sqrt{a^2 + y^2}$. It's clear from symmetry that the net force is in the y-direction, so we need to find only the y-components of the unit vectors. The y-components are clearly the same for each, and the drawing shows that they're given by $\hat{r}_y = y/\sqrt{a^2 + y^2}$.

EVALUATE From Coulomb's law, the y-component of the force from each q is $F_y = (kqQ/r^2)\hat{r}_y$, and the net force on Q becomes

$$\vec{F} = 2\left(\frac{kqQ}{a^2 + y^2}\right)\left(\frac{y}{\sqrt{a^2 + y^2}}\right)\hat{j} = \frac{2kqQy}{(a^2 + y^2)^{3/2}}\hat{j}$$

The factor of 2 comes from the two charges q, which contribute equally to the net force.

ASSESS Make sense? Evaluating $\vec{F}$ at $y = 0$ gives zero force. Here, midway between the two charges, Q experiences equal but opposite forces and the net force is zero. At large distances $y \gg a$, on the other hand, we can neglect a^2 compared with y^2, and the force becomes $\vec{F} = k(2q)Q\hat{j}/y^2$. This is just what we would expect from a single charge $2q$ a distance y from Q—showing that the system of two charges acts like a single charge $2q$ at distances that are large compared with the charge separation. In between our two extremes the behavior of force with distance is more complicated; in fact, its magnitude initially increases as Q moves away from the origin and then begins to decrease.

In drawing Fig. 20.7, we tacitly assumed that q and Q have the same signs. But our analysis holds even if they don't; then the product qQ is negative, and the forces actually point opposite the directions shown in Fig. 20.7.

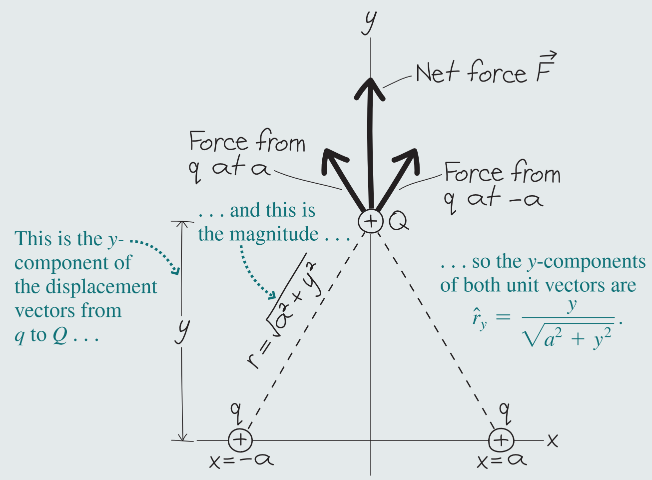


FIGURE 20.7 The force on Q is the vector sum of the forces from the individual charges.

20.3 The Electric Field

LO 20.4 *Describe the concept of electric field.*

In Chapter 8 we defined the gravitational field at a point as the gravitational force per unit mass that an object at that point would experience. In that context, we can think of $\vec{g}$ as the *force per unit mass* that any object would experience due to Earth's gravity. So we can picture the gravitational field as a set of vectors giving the magnitude and direction of the gravitational force per unit mass at each point, as shown in Fig. 20.8*a* below.

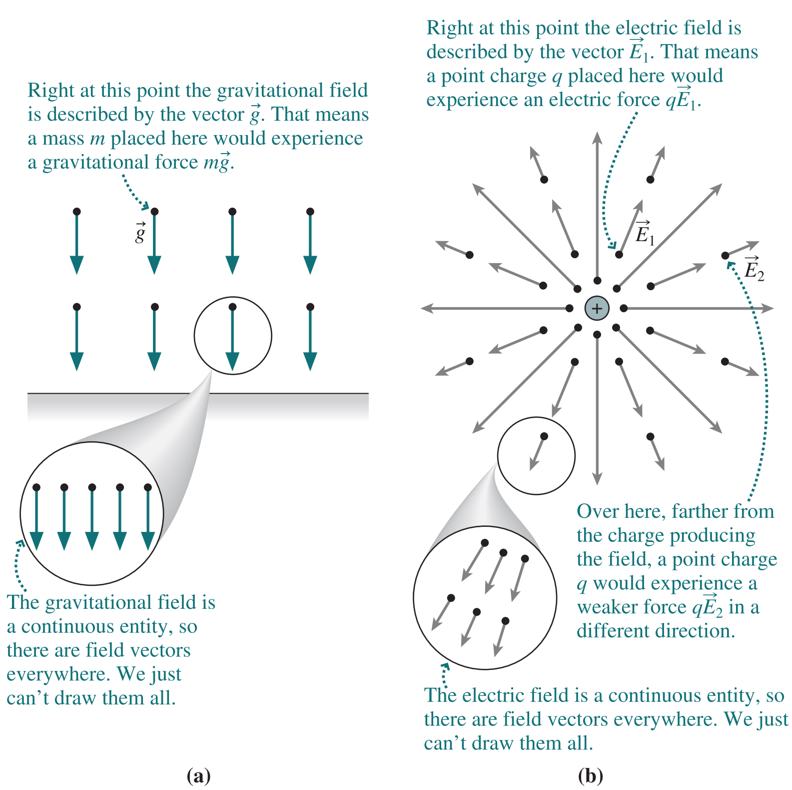
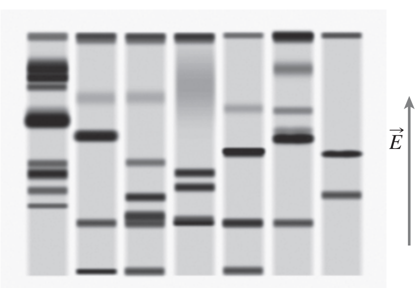


FIGURE 20.8 (a) Gravitational and (b) electric fields, here represented as sets of vectors.

APPLICATION **Electrophoresis**

Electrophoresis is a widely used application of electric fields for separating molecules by size and molecular weight. It's especially useful in biochemistry and molecular biology for distinguishing larger molecules like proteins and DNA fragments. In the commonly used *gel electrophoresis*, molecules carrying electric charge move through a semisolid but permeable gel under the influence of a uniform electric field; the greater the charge, the greater the electric force. The gel exerts a retarding force that increases with increasing molecular size, with the result that each molecular species moves at a velocity that depends on its size and charge. After a given time, the electric field is switched off. The locations of the molecules then serve as indicators of their size, with the molecules that traveled farthest being the smallest. The photo shows a typical gel electrophoresis result. Here DNA fragments were introduced into the seven channels at the top of the gel and then moved downward; their final locations indicate molecular size. The smaller molecules—those with fewer nucleotide base pairs—end up farther down on the gel. The electric field is shown by the arrow; it needs to point upward because DNA fragments carry a negative charge.



We can do the same thing with the electric force, defining the **electric field** as the force per unit charge:

> The electric field at any point is the force per unit charge that a charge would experience at that point. Mathematically,

$\vec{E}$ is the electric field at any point.

You can determine $\vec{E}$ by measuring the electric force $\vec{F}$ …

$$\vec{E} = \frac{\vec{F}}{q} \quad \text{(electric field)} \qquad (20.2a)$$

… on a small test charge q.

The electric field exists at every point in space. When we represent the field by vectors, we can't draw one everywhere, but that doesn't mean there isn't a field at all points. Furthermore, we draw vectors as extended arrows, but each vector represents the field at only one point—namely, the tail end of the vector. Figure 20.8*b* illustrates this for the electric field of a point charge.

The field concept leads to a shift in our thinking about forces. Instead of the action-at-a-distance idea that Earth reaches across empty space to pull on the Moon, the field concept says that Earth creates a gravitational field and the Moon responds to the field at its location. Similarly, a charge creates an electric field throughout the space surrounding it. A second charge then responds to the field at its immediate location. Although the field reveals itself only through its effect on a charge, the field nevertheless exists at all points, whether or not charges are present. Right now you probably find the field concept a bit abstract, but as you advance in your study of electromagnetism you'll come to appreciate that fields are an essential feature of our universe, every bit as real as matter itself.

We can use Equation 20.2a as a prescription for measuring electric fields. Place a point charge at some location, measure the electric force it experiences, and divide by the charge to get the field. In practice, we need to be careful because the field generally arises from some distribution of source charges. If the charge we're using to probe the field—the **test charge**—is large, the field it creates may disturb the source charges, altering their configuration and thus the field they create. For that reason, it's important to use a very small test charge.

If we know the electric field $\vec{E}$ at a point, we can rearrange Equation 20.2a to find the force on any point charge q placed at that point:

$\vec{F}$ is the electric force … … on a charge q …

$$\vec{F} = q\vec{E} \quad \text{(electric force and field)} \qquad (20.2b)$$

… at a point where the electric field is $\vec{E}$.

If the charge q is positive, then this force is in the same direction as the field, but if q is negative, then the force is opposite to the field direction.

Equations 20.2 show that the units of electric field are newtons per coulomb. Fields of hundreds to thousands of N/C are commonplace, while fields of 3 MN/C will tear electrons from air molecules. Sometimes we're interested in the magnitude of the field but not its direction. Then we can use Equations 20.2a and 20.2b without the vector signs. We'll often use the term *field strength* to be synonymous with the field's magnitude.

EXAMPLE 20.3 Force and Field: Inside a Lightning Storm

A charged raindrop carrying 10 μC experiences an electric force of 0.30 N in the $+x$-direction. What's the electric field at its location? What would the force be on a -5.0-μC drop at the same point?

INTERPRET In this problem we need to distinguish between electric force and electric field. The electric field exists with or without the charged raindrop present, and the electric force arises when the charged raindrop is in the electric field.

DEVELOP Knowing the electric force and the charge on the raindrop, we can use Equation 20.2a, $\vec{E} = \vec{F}/q$, to get the electric field. Once we know the field, we can use Equation 20.2b,

$\vec{F} = q\vec{E}$, to calculate the force that would act if a different charge were at the same point.

EVALUATE Equation 20.2a gives the electric field:

$$\vec{E} = \frac{\vec{F}}{q} = \frac{0.30\hat{\imath}\text{ N}}{10\ \mu\text{C}} = 30\hat{\imath}\text{ kN/C}$$

Acting on a -5.0-μC charge, this field would result in a force

$$\vec{F} = q\vec{E} = (-5.0\ \mu\text{C})(30\hat{\imath}\text{ kN/C}) = -0.15\hat{\imath}\text{ N}$$

ASSESS Make sense? The force on the second charge is opposite the direction of the field because now we've got a negative charge in the same field.

TIP **THE FIELD IS INDEPENDENT OF THE TEST CHARGE** Does the electric field in this example point in the $-x$-direction when the charge is negative? No. The field is independent of the particular charge experiencing that field. Here the electric field points in the $+x$-direction *no matter what charge* you put in the field. For a positive charge, the force $q\vec{E}$ points in the *same* direction as the field; for a negative charge, $q < 0$, the force is *opposite* the field.

The Field of a Point Charge

Once we know the field of a charge distribution, we can calculate its effect on other charges. The simplest charge distribution is a single point charge. Coulomb's law gives the force on a test charge q_{test} located a distance r from a point charge q: $\vec{F} = (kqq_{test}/r^2)\hat{r}$, where $\hat{r}$ is a unit vector pointing *away* from q. The electric field arising from q is the force per unit charge, or

$$\vec{E} = \frac{\vec{F}}{q_{test}} = \frac{kq}{r^2}\hat{r} \quad \text{(field of a point charge)} \qquad (20.3)$$

The electric field $\vec{E}$ is the force per unit charge.

For a point charge, $\vec{E}$ depends on the charge q …

… and on the distance r from the charge to the point where the field is being evaluated.

The unit vector $\hat{r}$ always points away from q, regardless of q's sign.

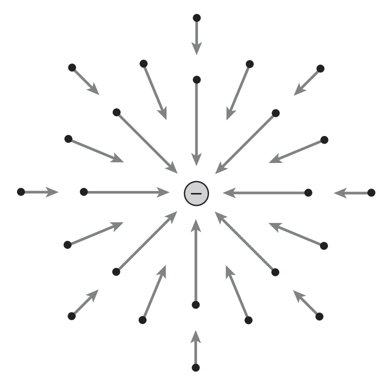


FIGURE 20.9 Field vectors for a negative point charge.

Since it's so closely related to Coulomb's law for the electric force, we also refer to Equation 20.3 as Coulomb's law. The equation contains no reference to the test charge q_{test} because the field of q exists independently of any other charge. Since $\hat{r}$ always points *away* from q, the direction of $\vec{E}$ is radially outward if q is positive and radially inward if q is negative. Figure 20.9 shows some field vectors for a negative point charge, analogous to those of the positive point charge in Fig. 20.8*b*.

GOT IT? **20.3** A positive point charge is located at the origin of an x–y coordinate system, and an electron is placed at a location where the electric field due to the point charge is given by $\vec{E} = E_0(\hat{\imath} + \hat{\jmath})$, where E_0 is positive. Is the direction of the force on the electron (a) toward the origin, (b) away from the origin, (c) parallel to the x-axis, or (d) impossible to determine without knowing the coordinates of the electron's position?

20.4 Fields of Charge Distributions

LO 20.5 *Determine the fields of electric charge distributions using superposition.*

LO 20.6 *Describe the electric dipole and the field it produces.*

LO 20.7 *Determine the fields of continuous charge distributions by integration.*

Since the electric force obeys the superposition principle, so does the electric field. That means the field of a charge distribution is the vector sum of the fields of the individual point charges making up the distribution:

The electric field $\vec{E}$ of a distribution of point charges… … is the sum of the fields of the individual point charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots = \sum_i \vec{E}_i = \sum_i \frac{kq_i}{r_i^2}\hat{r}_i \tag{20.4}$$

Here the $\vec{E}_i$'s are the fields of the point charges q_i located at distances r_i from the point where we're evaluating the field—called, appropriately, the **field point**. The $\hat{r}_i$'s are unit vectors pointing *from* each point charge *toward* the field point. In principle, Equation 20.4 gives the electric field of *any* charge distribution. In practice, the process of summing the individual field vectors is often complicated unless the charge distribution contains relatively few charges arranged in a symmetric way.

Finding electric fields using Equation 20.4 involves the same strategy we introduced for finding the electric force; the only difference is that there's no charge to experience the force. The first step then involves identifying the field point. We still need to find the appropriate unit vectors and form the vector sum in Equation 20.4. Example 20.4 shows how this is done.

Sometimes we're interested in finding not the electric field but a point or points where the field is zero. Conceptual Example 20.2 explores such a case.

EXAMPLE 20.4 Finding the Field: Two Protons

Two protons are 3.6 nm apart. Find the electric field at a point between them, 1.2 nm from one of the protons. Then find the force on an electron at this point.

INTERPRET We follow our electric-force strategy, except that instead of identifying the charge experiencing the force, we identify the field point as being 1.2 nm from one proton. The source charges are the two protons; they produce the field we're interested in.

DEVELOP Let's have the protons define the x-axis, as drawn in Fig. 20.10. Then the unit vector $\hat{r}_1$ from the left-hand proton toward the field point (which we've marked P) is $+\hat{\imath}$, while $\hat{r}_2$ from the right-hand proton toward P is $-\hat{\imath}$.

EVALUATE We now evaluate the field at P using Equation 20.4:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{ke}{r_1^2}\hat{\imath} + \frac{ke}{r_2^2}(-\hat{\imath}) = ke\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)\hat{\imath}$$

We wrote e for q here because the protons' charge is the elementary charge.

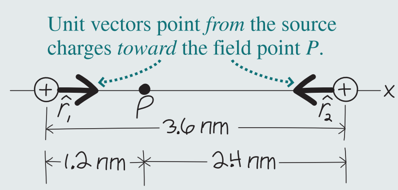


FIGURE 20.10 Finding the electric field at *P*.

Using $e = 1.6 \times 10^{-19}$ C, $r_1 = 1.2$ nm, and $r_2 = 2.4$ nm gives $\vec{E} = 750\hat{\imath}$ MN/C. An electron at P will therefore experience a force $\vec{F} = qE = -eE = -0.12\hat{\imath}$ nN.

ASSESS Make sense? The field points in the positive x-direction, reflecting the fact that P is closer to the left-hand proton with its stronger field at P. The force on the electron, on the other hand, is in the $-x$-direction; that's because the electron is negative (we used $q = -e$ for its charge), so the force it experiences is opposite the field. That field of almost 1 GN/C sounds huge—but that's not unusual at the microscopic scale, where we're close to individual elementary particles.

CONCEPTUAL EXAMPLE 20.2 Zero Field, Zero Force

A positive charge $+2Q$ is located at the origin, and a negative charge $-Q$ is at $x = a$. In which region of the x-axis is there a point where the force on a test charge—and therefore the electric field—is zero?

INTERPRET We're asked to locate qualitatively a point where the field is zero. Our sketch of the situation, Fig. 20.11, shows that the two charges divide the x-axis into three regions: (1) to the left of $2Q$ ($x < 0$), (2) between the charges ($0 < x < a$), and (3) to the right of $-Q$ ($x > a$). We need to determine which region could include a point where the electric force on a test charge is zero.

EVALUATE Consider what would happen to a positive test charge placed in each of these three regions. Anywhere in region (1), the test charge is closer to the charge with greater magnitude ($2Q$). That charge dominates throughout region (1), where our test charge would experience a repulsive force (to the left). The electric field, then, can't be zero in region (1). Between the two charges, the repulsive force from $2Q$ on a positive test charge points to the right; so does the attractive force from $-Q$. The field, therefore, can't be zero in region (2). That leaves region (3). Could the field be zero here? Put a positive test charge very close to $-Q$, and it experiences an attractive force toward the left. But far away, the distance between $2Q$ and $-Q$ becomes negligible. The fields of both charges drop off as the inverse square of the distance, so at large distances the field of the stronger charge will dominate. Therefore there *is* a point somewhere to the right of $-Q$ where the force on a test charge, and therefore the electric field, will be zero.

ASSESS This answer is consistent with our insight from Example 20.2 that when we get far from a charge distribution, it begins to resemble a point charge with the net charge of the distribution. Here that net charge is $2Q - Q = +Q$, so at large distances we should indeed have a field pointing away from the charge distribution—and that's to the right in region (3). Although we considered a positive test charge, you'll reach the same conclusion with a negative test charge.

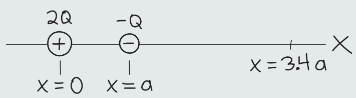


FIGURE 20.11 Where is the electric field zero? We've marked the answer, at $x = 3.4a$.

MAKING THE CONNECTION Find an expression for the position where the electric field in this example is zero.

EVALUATE In Fig. 20.11, we've taken the origin at $2Q$, so at any position x in region (3) we're a distance x from $2Q$ and a distance $x-a$ from $-Q$. Since we're to the right of both charges, the unit vector in Equation 20.3 for the point-charge field—a vector that always points away from the point charge—becomes $+\hat{\imath}$ for both charges. Applying Equation 20.3, $\vec{E} = (kq/r^2)\hat{r}$, for the fields of the two charges and summing gives

$$\vec{E} = \frac{k(2Q)}{x^2}\hat{\imath} + \frac{k(-Q)}{(x-a)^2}\hat{\imath}$$

If we set this expression to zero, we can cancel k, Q, and $\hat{\imath}$; inverting both sides of the remaining equation gives $x^2/2 = (x-a)^2$. Finally, taking the square root and solving for x gives the answer: $x = a\sqrt{2}/(\sqrt{2}-1) \simeq 3.4a$. As a check, note that this point does indeed lie to the right of $x = a$. We've marked this point in Fig. 20.11.

The Electric Dipole

One of the most important charge distributions is the **electric dipole**, consisting of two point charges of equal magnitude but opposite sign. Many molecules are essentially dipoles, so understanding the dipole helps explain molecular behavior (Fig. 20.12). During contraction the heart muscle becomes essentially a dipole, and physicians performing electrocardiography are measuring, among other things, the strength and orientation of that dipole. Antennas used in wireless communications—including radio, TV, wifi, and cellphones—are often based on the dipole configuration.

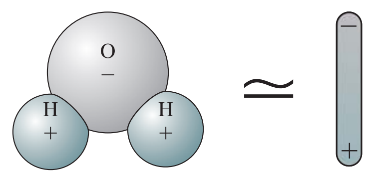


FIGURE 20.12 A water molecule behaves like an electric dipole. Its net charge is zero, but regions of positive and negative charge are separated.

EXAMPLE 20.5 The Electric Dipole: Modeling a Molecule

A molecule may be modeled approximately as a positive charge q at $x = a$ and a negative charge $-q$ at $x = -a$. Evaluate the electric field on the y-axis, and find an approximate expression valid at large distances ($|y| \gg a$).

INTERPRET Here's another example where we'll use our strategy in applying Equation 20.4 to calculate the field of a charge distribution. We identify the field point as being anywhere on the y-axis and the source charges as being $\pm q$.

DEVELOP Figure 20.13 is our drawing. The individual unit vectors point from the two charges toward the field point, but the *negative* charge contributes a field *opposite* its unit vector; we've indicated the individual fields in Fig. 20.13. Here symmetry makes the y-components cancel, giving a net field in the $-x$-direction. So we need only the x-components of the unit vectors, which Fig. 20.13 shows are $\hat{r}_{x-} = a/r$ for the negative charge at $-a$ and $\hat{r}_{x+} = -a/r$ for the positive charge at a.

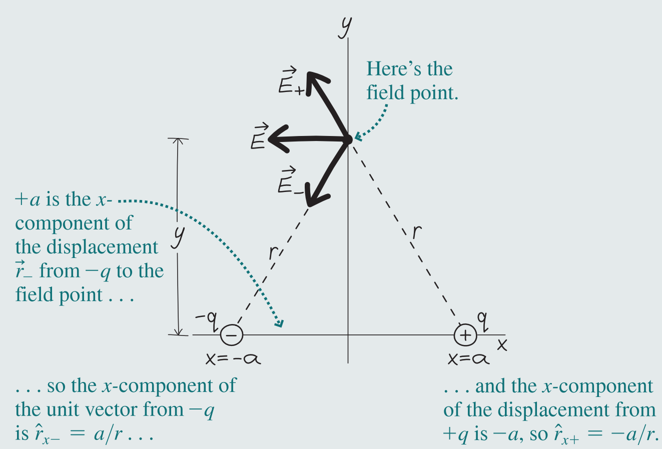


FIGURE 20.13 Finding the field of an electric dipole.

EVALUATE We then evaluate the field using Equation 20.4:

$$\vec{E} = \frac{k(-q)}{r^2}\left(\frac{a}{r}\right)\hat{\imath} + \frac{kq}{r^2}\left(-\frac{a}{r}\right)\hat{\imath} = -\frac{2kqa}{(a^2+y^2)^{3/2}}\hat{\imath}$$

where in the last step we used $r = \sqrt{a^2 + y^2}$. For $|y| \gg a$ we can neglect a^2 compared with y^2, giving

$$\vec{E} \simeq -\frac{2kqa}{|y|^3}\hat{\imath} \qquad (|y| \gg a)$$

(*continued*)

ASSESS Make sense? The dipole has no net charge, so at large distances its field can't have the inverse-square drop-off of a point-charge field. Instead the dipole field falls faster, here as $1/|y|^3$. Note that we were careful to put absolute value signs on y^3; that way, our result applies for both positive and negative values of y.

TIP **APPROXIMATIONS** Making approximations requires care. Here we're basically asking for the field when y is so large that a is negligible *compared with* y. So we neglect a^2 compared with y^2 when the two are summed, but we *don't* neglect a when it appears in the numerator, where it isn't being directly compared with y.

Example 20.5 shows that the dipole field at large distances decreases as the inverse *cube* of distance. Physically, that's because the dipole has zero *net* charge. Its field arises entirely from the slight separation of two opposite charges. Because of this separation, the dipole field isn't exactly zero, but it's weaker and more localized than the field of a point charge. Many complicated charge distributions exhibit the essential characteristic of a dipole—that is, they're neutral but consist of separated regions of positive and negative charge—and at large distances, such distributions all have essentially the same field configuration.

At large distances the dipole's physical characteristics q and a enter the equation for the electric field only through the product qa. We could double q and halve a, and the dipole's electric field would remain unchanged. At large distances, therefore, a dipole's electric properties are characterized completely by its **electric dipole moment** p, defined as the product of the charge q and the separation d between the two charges making up the dipole:

$$p = qd \quad \text{(dipole moment)} \tag{20.5}$$

In Example 20.5 the charge separation was $d = 2a$, so there the dipole moment was $p = 2aq$. In terms of the dipole moment, the field in Example 20.5 can then be written

$$\vec{E} = -\frac{kp}{|y|^3}\hat{\imath} \quad \left(\begin{array}{l}\text{dipole field for } |y| \gg a,\\ \text{on perpendicular bisector}\end{array}\right) \tag{20.6a}$$

You can show in Problem 54 that the field on the dipole axis is given by

$$\vec{E} = \frac{2kp}{|x|^3}\hat{\imath} \quad \left(\begin{array}{c}\text{dipole field}\\ \text{for } |x| \gg a, \text{ on axis}\end{array}\right) \tag{20.6b}$$

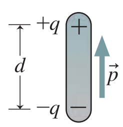


FIGURE 20.14 The dipole moment vector $\vec{p}$ has magnitude $p = qd$ and points from the negative toward the positive charge.

Because the dipole isn't spherically symmetric, its field depends not only on distance but also on orientation; for instance, Equations 20.6 show that the field along the dipole axis at a given distance is twice as strong as along the bisector. So it's important to know the orientation of a dipole in space, and therefore we generalize our definition of the dipole moment to make it a vector of magnitude $p = qd$ in the direction from the negative toward the positive charge (Fig. 20.14).

GOT IT? **20.4** Far from a charge distribution, you measure an electric field strength of 800 N/C. What will the field strength be if you double your distance from the charge distribution, if the distribution consists of (1) a point charge or (2) a dipole?

Continuous Charge Distributions

Although any charge distribution ultimately consists of pointlike electrons and protons, it would be impossible to sum all the field vectors from the 10^{23} or so particles in a typical piece of matter. Instead, it's convenient to make the approximation that charge is spread continuously over the distribution. If the charge distribution extends throughout a volume, we describe it in terms of the **volume charge density** ρ, with units of C/m^3. For charge distributions spread over surfaces or lines, the corresponding quantities are the **surface charge density** σ (C/m^2) and the **line charge density** λ (C/m).

To calculate the field of a continuous charge distribution, we divide the charged region into very many small charge elements dq, each small enough that it's essentially a point charge. Each dq then produces an electric field $d\vec{E}$ given by Equation 20.3: $d\vec{E} = (k\,dq/r^2)\hat{r}$. We then form the vector sum of all the $d\vec{E}$'s (Fig. 20.15). In the limit of infinitely many infinitesimally small dq's and their corresponding $d\vec{E}$'s, that sum becomes an integral and we have

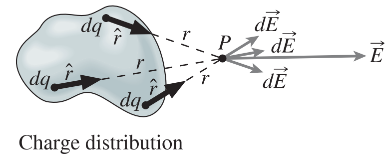


FIGURE 20.15 The electric field at P is the vector sum of the fields $d\vec{E}$ arising from the individual charge elements dq, each calculated using the appropriate distance r and unit vector $\hat{r}$.

The electric field $\vec{E}$ of a continuous distribution of charges …

dq is an infinitesimal charge element.

$\hat{r}$ is a unit vector that points away from dq, regardless of its sign.

$$\vec{E} = \int d\vec{E} = \int \frac{k\,dq}{r^2}\hat{r} \quad \left(\begin{array}{c}\text{field of a continuous}\\ \text{charge distribution}\end{array}\right) \tag{20.7}$$

… is determined by integrating the fields $d\vec{E}$ of infinitesimal charge elements dq.

r is the distance from dq to the point where the field is being evaluated.

The limits of this integral include the entire charge distribution.

Calculating the field of a continuous charge distribution involves the same strategy we've already used: We identify the field point and the source charges—although now the source is a continuous charge distribution. Summing the individual field contributions now presents us with an integral, and that means writing the unit vectors $\hat{r}$ and distances r in terms of coordinates over which we can integrate. Setting up the integral involves the same strategy we outlined in Chapter 9 to find the center of mass of a continuous distribution of matter, and used again in Chapter 10 to find rotational inertias.

EXAMPLE 20.6 Evaluating the Field: A Charged Ring

A ring of radius a carries a charge Q distributed evenly over the ring. Find an expression for the electric field at any point on the axis of the ring.

INTERPRET We identify the field point as lying anywhere on the ring's axis, and the source charge as the entire ring.

DEVELOP Let's take the x-axis to coincide with the ring axis, with the center of the ring at $x = 0$ (Fig. 20.16). The figure shows that the y-components of the field contributions from pairs of charge elements on opposite sides of the ring cancel; therefore, the net field points in the $+x$-direction (for $x > 0$) and we need only the x-components of the unit vectors. Those are the same for all unit vectors—namely, $\hat{r}_x = x/r$.

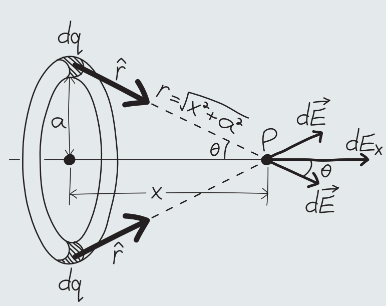


FIGURE 20.16 The electric field of a charged ring points along the ring axis, since field components perpendicular to the axis cancel in pairs.

EVALUATE We're now ready to set up the integral in Equation 20.7. Here each charge element contributes the same amount $dE_x = (k\,dq/r^2)\hat{r}_x = (k\,dq/r^2)(x/r)$ to the field. Figure 20.16 shows that $r = \sqrt{x^2 + a^2} = (x^2 + a^2)^{1/2}$, so the integral becomes

$$E = \int_{\text{ring}} dE_x = \int_{\text{ring}} \frac{kx\,dq}{(x^2 + a^2)^{3/2}} = \frac{kx}{(x^2 + a^2)^{3/2}} \int_{\text{ring}} dq$$

The last step follows because we have a fixed field point P, so its coordinate x is a constant for the integration. But the remaining integral is just the sum of all the charge elements on the ring—namely, the total charge Q. So our result becomes

$$E = \frac{kQx}{(x^2 + a^2)^{3/2}} \quad \text{(on-axis field, charged ring)}$$

This is the magnitude; the direction is along the x-axis, away from the ring if Q is positive and toward it if Q is negative.

ASSESS Make sense? At $x = 0$ the field is zero. A charge placed at the ring center is pulled (or pushed) equally in all directions—no net force, so no electric field. But for $x \gg a$, we get $E = kQ/x^2$—just what we expect for a point charge Q. As always, a finite-sized charge distribution looks like a point charge at large distances. Problem 73 shows how you can use the result of this example to find the electric field on the axis of a charged disc, and Problem 75 shows that, once again, the field at large distances becomes that of a point charge.